Unit -II Section B.

(#1 Unit-II Section B)

Superconductivity

Q.1 a) what is superconductivty
b) Give their main characterstic properties

Ans :- a) The phenomenon in which the electric resistivity of a material suddenly falls to 'zero' when the material is cooled to a sufficient low temperature is called as "superconductivity" and the material is called as a "super conductor". It was discovered by Heike Onnes with his experiments with liquid He. as shown in fig(a) The resistance of Hg suddenly falls to zero. at about 4.2 K

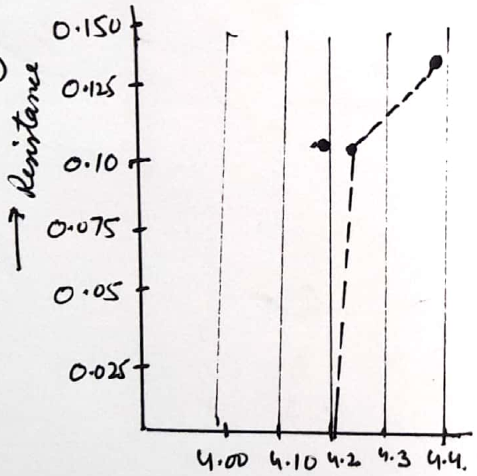


fig (a)

· Critical Temperature (T_c) of a material is defined as the temperature its resistance suddenly falls to zero. It is also called as super conducting Transition temperature.

b) Characterstic Properties

1) At critical temperature its conductivity falls to zero

2) If the external magnetic field employed to cool below T_c is switched off. it is found that the induced current in the material presists in it for a long duration and decays as per $i = I_0 e^{-\frac{R}{L}t}$ it is called as "presistent current"

3) When placed inside a magnetic field a super conductor material behaves as a perfect diamagnet.

4) If the externally applied magnetic field is strong it may destroy superconductivity at "Critical field H_c"

5) Even if the amount of current through a superconducting material is larger than its critical value then also superconductivity is destroyed.

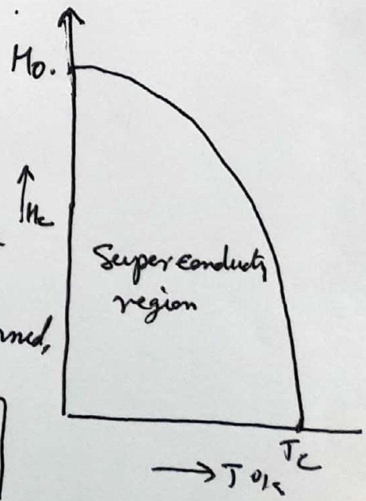
6) Properties of superconductors show a change in properties when current through it is of very high frequency (>10 MHz)

7) Introduction of impurities affects the superconductivof properties

8) when placed under stress the temperature of super conductor rises $\left(= \frac{dT_c}{dP}\right)$. This also affects the superconductivity of the material.

9) Important

The critical magnetic field (H_c) is defined as that magnetic field where half the normal resistance has returned, From graph.



$$H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

At 0K $H_c = H_0$ & $T = T_c$, $H_c = 0$.

Q.2 Explain and distinguish b/w Type-I & Type-II super-conductors

Ans. Depending upon the magnetic behaviour in an external magnetic field. These ~~are~~ have two types

1. Type-I or soft superconductors

These are perfectly dimagnetic & exhibit Meissner effect completely

From the graph b/w $\vec{H}$ applied magnetic Field & $-\mu_0 M$ → diamagnetism it is observed that

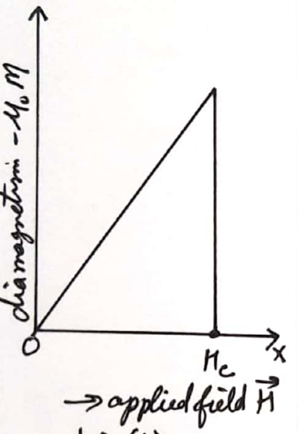


fig (b)

a) The superconducting material is "perfect dimagnetic" below the critical magnetic field value (H_c)

b) The material looses its superconducting property suddenly at H_c and diamagnetism suddly dies out i.e the magnetic field lines enter the specimen.

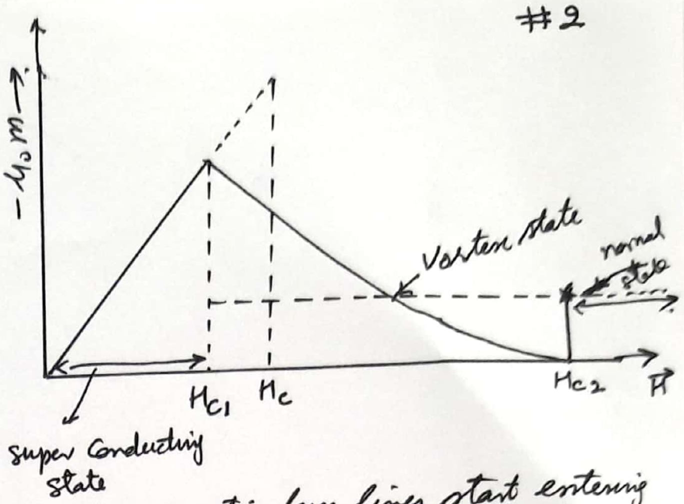
c) Above H_c it behaves as a normal conductor

Conclusion

a) As the value of H_c is too small. hence do not have much importance in applications.

b) These are generally pure samples of ~~the~~ the materials.

2. Type-II In this case the graph b/w $\vec{H}$ and $-\mu_0 M$ is more elaborate. (fig c)



a) The magnetic flux lines start entering the material at H_{c1} at a value less than H_c.

b) The material is said to be in Vortex state b/w H_{c1} & H_{c2}

c) In the vortex state $B \neq 0$ and Meissner effect is incomplete.

d) Value of H_{c2} may be 100 times that of H_c.

e) In Vortex state the material has properties of both superconductor & ordinary material.

f) Beyond H_{c2} it is a normal state.

These materials are generally alloys of transition metals with high value of electrical resistance in normal state.

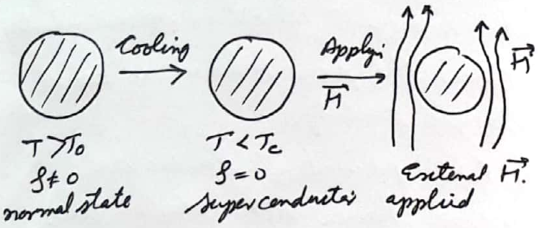
These materials are useful in tecnical applications.

Imp A close examination of specimen shows small ~~regions in~~ regions in normal state called as Vortices surrounded by large regions of super conducting ~~state~~. regions.

Q3 What is Meissner effect

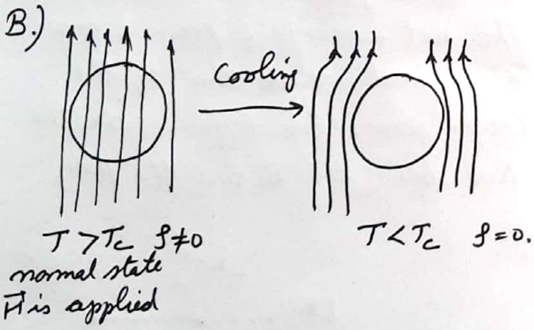
Ans:- The effect was observed by Meissner and Ochsenfield in 1933. The observations were made in two ~~diss~~ distinct ways

A)



In the first case a specimen is made superconducting by cooling it st $T < T_0$. Then applying the external magnetic field. wherein it exibits the property of diamagnetism ie ~~so~~ no flux lines entre the specimen

B.)



In this second case the specimen in normal state is placed in a magnetic field and as such the flux ~~lies~~ lines pass through it. But as the temperature of specimen is brought down st $T < T_c$ it becomes a superconductor and flux lines are pushed out of the specimen and it starts behaving as a perfect diamagnetic material.

Mathematically

$$B = \mu_0 H + M$$

magnetic Induction

Applied field

magnetisation produced inside specimen.

At superconducting stage

$B = 0. \Rightarrow \vec{M} = -\vec{H}.$

Now $\chi = \frac{\vec{M}}{\vec{H}} = -1$ (magnetic susceptibility)

= perfect diamagnet.

Let in normal state. the current density be j and $\rho \rightarrow$ resistivity

From Ohms law in microscopic form.

$$\vec{E} = \rho j$$

At temp $T < T_0$. superconducting stage

$\rho = 0$ & if j is finite (residual current)

$\Rightarrow E = 0$

Applying Maxwell's eq^n.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

as $E = 0 \Rightarrow \frac{\partial B}{\partial t} = 0.$ $B =$ constant.

i.e it should not change on cooling but as observed in second case. it is not so.

Thus from

a) Zero resistivity $\vec{E} = 0$

& b) from Meissner effect $\vec{B} = 0$

coexist in a superconductor

Do. Thermodynamic Properties of super conductors from book.

1) Specific Heat
2) Entropy
3) Thermal conductivity

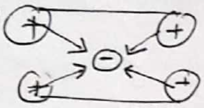
AND

Also do uses of semiconductors.

BCS the Bardeen, Cooper, Schreiffer Theory of superconductivity

As per the BCS theory the basic interaction b/w pair of e^-'s via the interchange of virtual photons is responsible for the superconductivity in the material. as explained in the following steps

1) when an e^{-1} approaches a +ve ion core it undergoes attractive coulomb's interaction therby setting the ion core in motion which in turn distorts the lattice.



~~2)~~

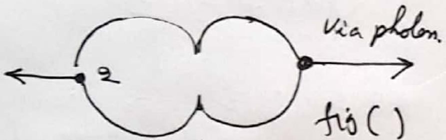


fig ( )

2. When a second e^{-1} approaches it interacts with distorted lattice s.t the energy of 2nd e^{-1} is lowered.

3. This above applies that the e^{-1}'s interact via lattice distortion or "photon field" which lowers the energy of the e^{-1}'s. It is also called as electron-lattice-electron interaction.

4. Thus it appears that e^{-1}'s 1 & 2 are bound via the generation of phonon.

5. The two bound e^-'s must have opposite spins. or opposite linear momentum s.t net momenta of the pair is zero. fig( )

6. Due to phonon exchang the interaction energy is −ve. or attractive force.

7) Now at lower temperature ~~to~~ this attractive force b/w pair of electrons exceeds the coulomb repulsion.

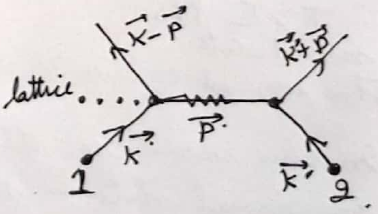
8) This leads to the formation of loosly bound state b/w electrons called as "COOPER PAIR" having an energy $\simeq 10^{-3}$ eV in super conductors. Thus the pair behaves as ~~an~~ a single entity.

9) The paired e^-'s forms as BOSON. hence net spin is zero. Called as "Cooper pairs"

10) When such pairs are produced the property of superconductivity is manifested. The pairing being completed at $T = 0K$ and completely broken at $T \geqslant T_C$. During conduction these pairs do not interact with lattice s.t resistance. zero

11) Binding energy

The net energy of pair of free e^{-1} is greater than that of the Cooper pair. The difference b/w the two being the B·E of the pair.



let e^{-1} 1 ~~interacts~~ having wave vector $\vec{K}$ interacts with lattice to produce virtual phonon of momentum $\vec{P}$. This is absorbed by e^{-1} 2 having wave vector K'. Thus after interaction

$\vec{K}-\vec{P} \rightarrow$ momentum of 1st e^{-1}.
$\vec{K'}+\vec{P} \rightarrow$ " " 2nd e^{-1}.

This involves the exchange of "Virtual phonon" it is so called because it is a very ~~slo~~ short time interaction hence energy is not conserved.

Defects in crystals.

The defects in natural crystals arise due to the conditions under which they are crystalized.

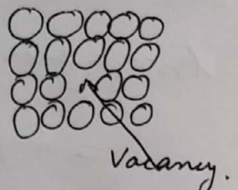
Defects are also called as imperfections

Crystalline defects can be classified on basis of their geometry as

1) Point Defects 2) Line defects
3) Surface Defects & 4) Volume defects

· Point defects

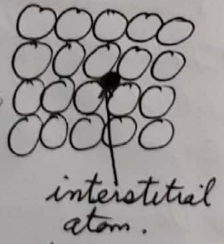
a). Vacancies :-



~~It~~ In this case atom is missing from the crystal giving rise to a vacancy.

It arises due to imperfection of packing in a crystal.

b). Interstitial Defects.

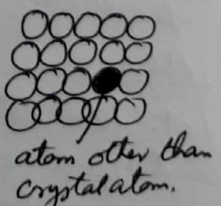


It refers to a condition when an external atom is found in voids of a regular crystal.

Such an atom is smaller in size than the atoms of the crystal.

eg in FCC iron crystal $r = 0.225$ nm is occupied by C-atom $r = 0.077$ nm

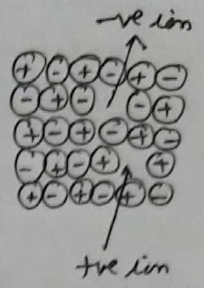
c). Subritutianal defect



It happens when one of the atoms of crystal gets replaced by a foreign atom.

This defect is deliberately introduced in case of Extrinsic Semiconductors wherein a trivalent atom is introduced in intrinsic (pure) semiconductor to form n-type.
& a penta valent atom is introduced to form p-type semiconductors.

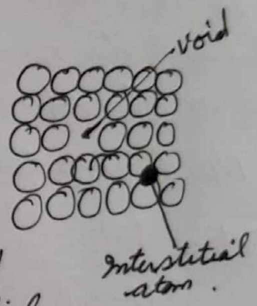
d) Schottky Defect



Whenever an equal number of +ve & -ve ions are missing in a crystal lattice. It is called as Schottky defect.

The crystal is electrical neutral. These defects are ~~abanda~~ prevalent in Alkali halides, NaCl, CsCl etc.

e) Frankel Defect



In this type of defect the crystal again remains ~~so~~ electrically neutral as an ion leaves its actual position in crystal and occupies an interstitial position. This defect is a combination of a "Vacancy" & "Interstitial" defect. These are present in Ag halides.

~~Do~~ difference b/w "Schottky" and "frankel" defects

Schottky Defect

i) It resembles a vacancy defect.
ii) These are formed when atoms or ions move towards the surface of the crystal. Thereby leaving voids in actual crystal lattice.
iii) The crystals having this defect are less dense
iv) Mostly found in alkali halides

Frankel Defect

i) Related to both "vacancy" & "interstitial" defects
ii) An atom/ion leaves its regular position & occupies an interstitial position.
iv) Density of crystal remains same.
v) Mostly found in Silver halides. eg AgBr.

Q Determine Equlibrium number of
a) Frankel defect &
b) Schottky defect.

Ans
(a) The disorder / defect increases with temperature, as energy is required to displace ~~past~~ a atom) ion from its regular position ~~on the surface of the crystal~~.

At equlibrium
Let $N \to$ total ions in a crystal & having $n' \to$ no of interstitial atoms. or $n' \to$ ~~not~~ no. of vacancies.

No. of ways the (n_f) defect can occur:

$$W = \frac{N!}{(N-n_f)!\,n_f!} \cdot \frac{n'}{(n'-n_f)!\,n_f!} \quad -(1)$$

Increase in entropy

$$S = k\ln W = k\ln\left[\frac{N!}{(N-n_f)!} \cdot \frac{n'}{(n'-n_f)!\,n_f!}\right] \quad -(2).$$

If E_i is energy requred to displace an atom from its regular postion to interstitial position. Then for n' no of Frankel defects, Energy requrid will be

$$U = n' E_i$$

and change in Helmholtz Energy.

$$F = U - TS$$

$$= n_f E_i - k\ln\left[\frac{N!}{(N-n_f)!} \cdot \frac{n'}{(n'-n_f)!\,n_f!}\right] \quad -(3).$$

Applying Sterlings Relation (Expansion) ($\ln x! = x\ln x - x$) (solving bracket)

$$\ln\left[\frac{N!}{(N-n_f)!\,n_f!}\right] + \ln\left[\frac{n'!}{(n'-n_f)!\,n_f!}\right]$$

$$F = n_f E_i = N\ln N + n'\ln n' - (N-n_f)\ln(N-n_f) - (n'-n_f)\ln(n'-n_f) - 2n_f\ln n_f$$

At $\equiv m$ $\left(\frac{\partial F}{\partial n}\right)_T = 0.$

$$E_i - kT\left[\ln(N-n_f) + \ln(n'-n_f) - 2\ln n_f\right] = 0.$$

$$E_i = kT\ln\left[\frac{(N-n_f)(n'-n_f)}{n_f^2}\right]$$

$$\Rightarrow \frac{E_i}{kT} = \ln\left[\frac{(N-n_f)(n'-n_f)}{n_f^2}\right]$$

applying approx.

$n_f << N$ & $n_f << n'$

$\Rightarrow (N-n_f) \simeq N$ & $(n'-n_f) \simeq n'$

$$\frac{E_i}{kT} = \ln\left[\frac{Nn'}{n_f^2}\right]$$

$$\ln(Nn') - 2\ln n_f = \frac{E_i}{kT}.$$

determination of n_f by transposing

$$\ln n_f = \frac{1}{2}\ln(Nn') - \frac{E_i}{2kT}.$$

$$= \ln(Nn')^{1/2} + \ln e^{-E_i/2kT}.$$

$$\ln n_f = \ln (Nn')^{\frac{1}{2}} e^{-E_i/2kT}$$

or $\boxed{n_f = (Nn')^{1/2}\exp\left(-\frac{E_i}{2kT}\right)}$

Thus the defect rises / increased with increase of temperature exponentially.

(b) Density / number of Schottky defect at $\equiv m$

As energy is requred to pull an atom / ion from its regular lattice position to surface of crystal hence greater the temperature greater will be the number of defects produced.

At $\equiv m$

Let. $N \to$ total ions of a crystal.
& $n_s \to$ no of scattky defects.
(ie n_s no. of cation & anion defects)

No. of ways the defects can be produced

$$W = \left[\frac{N!}{(N-n_s)!\,n_s!}\right]^2 \quad -(i)$$

factor of '2' as it is produced in pairs.

Due to defects produced the increase in entropy is given by

$$S = \ln W = k \ln\left[\frac{N!}{(N-n_s)!\,n_s!}\right] \quad \text{— (ii).}$$

Also total energy required to produce n_s no of defect if E_i is energy required to produce a pair of vacancies

$$U = n_s E_i$$

& Helmholtz energy

$$F = U - TS.$$

$$= nE_i - kT\ln\left[\frac{N!}{(N-n_s)!\,n_s!}\right]$$

$$= nE_i - kT\left[\ln N! - \ln(N-n_s)! - \ln n_s!\right]$$

Using Sterling formula.

$$\ln x! = x\ln x - x.$$

$$F = nE_i - 2kT\left[N\ln N - N - (N-n_s)\cancel{\ln}\,\ln(N-n_s) - n_s\ln n_s - n_s + (N-n)\right]$$

$$= nE_i - \cancel{n}\,2kT\left[N\ln N - (N-n_s)\ln(N-n_s) - n_s\ln n_s\right]$$

At ≡m

$$\left(\frac{\partial F}{\partial n_s}\right) = 0.$$

$$\Rightarrow E_i - 2kT\left[0 + \ln(N-n_s) + 1 - \ln n_s - 1\right] = 0.$$

$$E_i = 2kT\ln\left[\frac{N-n_s}{n_s}\right]$$

determination of n_s by transposing

$$\ln\left[\frac{N-n_s}{n_s}\right] = \frac{E_i}{2kT}$$

taking antilog.

$$\frac{N-n_s}{n_s} = e^{E_i/2kT}.$$

as $n_s << N \Rightarrow N - n_s \simeq N$

$$\frac{N}{\quad} = \exp\left(E_i/2kT\right)$$

$$\boxed{\text{or } n_s = N\exp\left(-E_i/2kT\right)}$$

NOTE

Students should note that in both cases the only difference is that in **W**. Rest of the steps are the same

Usefulness of the two defects

Both the defects play a vital role in the use/application of the crystals such as

i) These alter the optical property

ii) These alter the magnetic property

iii) ~~They~~ These defects reduces/obstructs the mobility of ions.

iv) These defects can act as acceptors and donor points.

v) These alter the mechanical properties.